

ECON G115: MATHEMATICAL GAME THEORY

Item	Value
Top Code	220400 - Economics
Units	3 Total Units
Hours	54 Total Hours (Lecture Hours 54)
Total Outside of Class Hours	0
Course Credit Status	Credit: Degree Applicable (D)
Material Fee	No
Basic Skills	Not Basic Skills (N)
Repeatable	No
Open Entry/Open Exit	No
Grading Policy	Standard Letter (S), • Pass/No Pass (B)
Local General Education (GE)	• Area 2 Mathematics (GB2)

Course Description

Formerly: Game Theory. This course provides an introduction to the mathematics of game theory. The course will explore game strategies and interactions between rational decision makers. It will use mathematical techniques and probabilities to calculate and maximize outcomes of multiplayer games. The course will build game theory frameworks to apply to games including Prisoner's Dilemma, games of chance, strategic games, and real world applications of market competition and international relations. PREREQUISITE: Course taught at the level of intermediate algebra or appropriate math placement. Transfer Credit: CSU; UC.

Course Level Student Learning Outcome(s)

1. Course Outcomes
2. Solve linear systems for mixed strategy probabilities.
3. Use game matrices and game trees to solve for optimal decisions of multiplayer games.
4. Apply game theory strategies to real world decision making.

Course Objectives

- 1. Describe mathematical techniques used in game theory decision making.
- 2. Use algebraic techniques, including linear models and systems of equations, in game theory applications.
- 3. Solve for the probability and expected value of outcomes in game theory applications.
- 4. Analyze quantitative solutions in game strategies to solve for the maximized outcome.
- 5. Identify strategies and equilibrium in game theory matrices.
- 6. Describe players and payoffs in vectors.
- 7. Solve zero-sum and non-zero-sum games for optimal strategies and equilibrium.
- 8. Calculate the expected value of mixed strategy games and solve for the optimal strategy.
- 9. Transform matrices to extensive form game trees.

- 10. Apply backward induction to solve for Subgame Perfect Nash Equilibrium in sequential games.
- 11. Apply the fixed point theorem in the proof of the existence of Nash equilibrium.
- 12. Apply the concepts of game theory to mechanism design.

Lecture Content

Game Matrices and Payoff Vectors Players Matrices 2x2 games 2x3 games Larger matrices Symmetric versus asymmetric Compute Payoff Matrices Evaluate numerical options to determine payoffs in a matrix Calculate probabilities of outcomes Calculate all possible outcomes Payoffs: $u_i > 0, u_{ii} = 0$ Sets, Probability, and Expected Values Sets and subsets of decisions Evaluate elements in sets $A \cup B, A \cap B$ Subsets Combinations Solve for the number of combinations of n taken r at a time: $nCr = n! / r!(n-r)!$ Permutations Solve for the number of permutations of n taken r at a time: $nPr = n! / (n-r)!$ Trees/Extensive Form Games Decision nodes Calculate the number of possible outcomes in a decision tree Solve for the expected probability of each outcome: $P(x) = n(E)/n(S)$ Compare the quantitative outcomes at each node of the decision tree to solve for the optimal strategy Subgames Games of Chance Solve for the probability of outcomes and the expected value of payoff: $EV = \sum P(x)x$ Game applications Rolling dice Roulette Poker Compare actual outcomes to expected outcomes as sample size increases Perfect Information Preferences with uncertain outcomes Equilibrium Nash/Best Response Solve for the best response of each player against other players simultaneously Maximize gains of player i

Lab Content

all other players Strategy profile s for every player i : $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$ Fixed point theorem Bayesian Conditional probability of event X based on occurrence of event Y Bayes Theorem: $P(X|Y) = \frac{P(X)P(Y|X)}{P(Y)}$

Method(s) of Instruction

- Lecture (02)
- DE Live Online Lecture (02S)
- DE Online Lecture (02X)

Instructional Techniques

$Y) = \frac{P(X)P(Y|X)}{P(Y)}$

Reading Assignments

$X)P(X) / (P(Y))$ Pair Deadlock Selten/subgame perfect Strategies Pure Zero randomization Dominant Strictly dominant $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$ Weakly dominant Mixed Equations of expected value using payoff matrix Unknown variables Algebraic substitution Mixed strategy algorithm Probability of mixed strategy Efficiency Graphical Solutions Equation of a Line: $y = mx + b$ Systems of Equations Algebraic Substitution Types of Games Zero-sum games Non-zero sum games Discrete game Continuous rounds Simultaneous decisions Sequential decisions First move advantage Iterated Elimination Backward Induction Analyze numerical payoffs of a game from back to front Calculate possible outcomes of decision tree nodes Analyze quantitative outcomes to describe the optimal strategy at each node of a decision tree Solve for the optimal opening strategy that maximizes expected gains for each player Solve for equilibrium Prisoner's Dilemma Cooperation Tit for Tat Nash equilibrium: $u^* = ((0,1), (0,1))$ Sample Games Matching Pennies Chicken Tic Tac Toe Rock-Paper-Scissors Second price auction Applications Price

fixing Price wars Duopolistic competition Quid pro quo International trade and tariffs Arms races/escalation Mechanism design

Demonstration of Critical Thinking

Problem solving exercises related to game theory applications and decisions Written solutions to homework problems Students will calculate and solve math problems related to games, and decision making strategy Use statistical techniques including probability and expected value to solve for maximize outcomes for game strategies.

Required Writing, Problem Solving, Skills Demonstration

Written solutions to homework, quiz, and test problems. Quantitative analysis of mathematical problem solving including an analysis of the outcomes.

Eligible Disciplines

Economics: Master's degree in economics OR bachelor's degree in economics AND master's degree in business, business administration, business management, business education, finance, or political science OR the equivalent. Master's degree required. Mathematics: Master's degree in mathematics or applied mathematics OR bachelor's degree in either of the above AND master's degree in statistics, physics, or mathematics education OR the equivalent. Master's degree required.

Textbooks Resources

1. Required Nordstrom, J. Introduction to Game Theory: a Discovery Approach (Classic), ed. Jennifer Firkins Nordstrom (OER), 2020 2. Required Bonanno, G. Game Theory, 3rd ed. Giacomo Bonanno (OER), 2024 Rationale: . 3. Required Spaniel, W. Game Theory 101: The Complete Textbook (Classic), ed. William Spaniel, 2020 4. Required Faigle, Ulrich. Mathematical Game Theory, ed. World Scientific Publishing, 2022